



EE 232 Lightwave Devices

Lecture 14:

Strained Quantum Well Laser

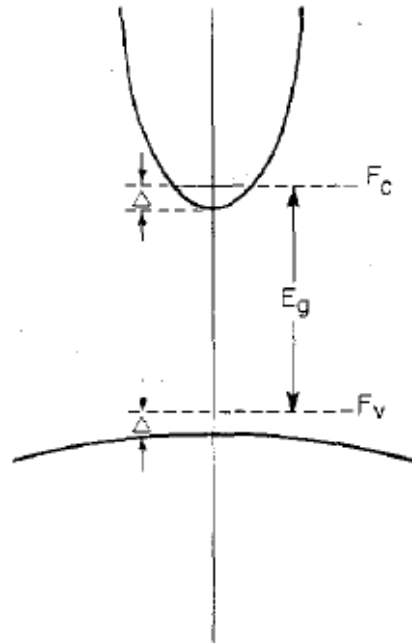
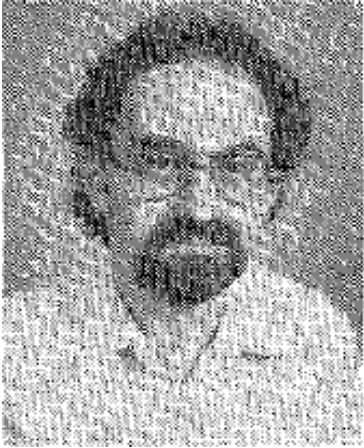
Reading: Chuang, Sec. 10.3-10.4
(There is also a good discussion in Coldren, Appendix 11)

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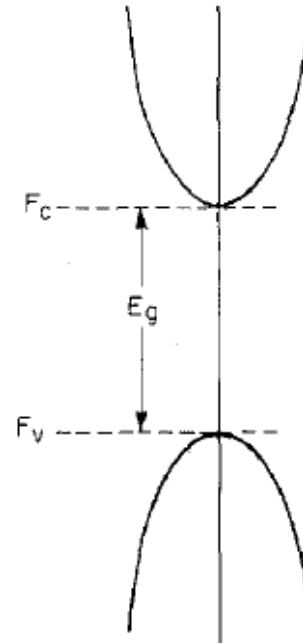
Reduction of Lasing Threshold Current Density by Lowering Valence Band Effective Mass



Ordinary Semiconductor

$$m_h^* \approx 6m_e^*$$

High transparency
carrier concentration



Ideal Semiconductor

$$m_h^* \approx m_e^*$$

Low transparency
carrier concentration

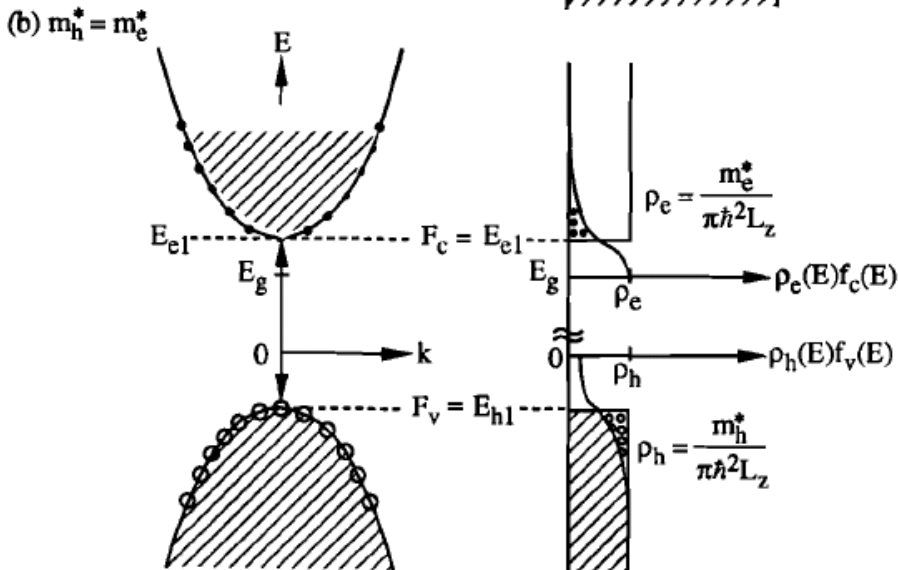
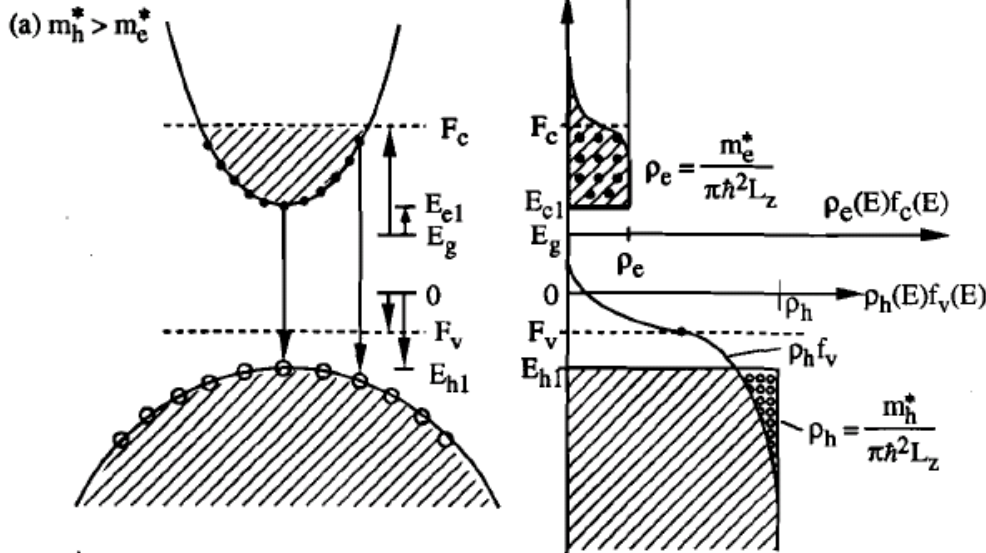
Bernard-Duraffourg Condition:

$$F_C - F_V \geq \omega \geq E_{e1} - E_{h1}$$

- Yablonovitch, E.; Kane, E., "Reduction of lasing threshold current density by the lowering of valence band effective mass," *Lightwave Technology, Journal of*, vol.4, no.5, pp. 504-506, May 1986
- Yablonovitch, E.; Kane, E.O., "Band structure engineering of semiconductor lasers for optical communications," *Lightwave Technology, Journal of*, vol.6, no.8, pp.1292-1299, Aug 1988



Bernard-Duraffourg Condition in Quantum Well



Bernard-Duraffourg Condition:

$$F_C - F_V = E - E_{h1}$$

(a) $m_h^* > m_e^*$ (as in most semiconductors)

$$F_V > E_{h1}$$

$$F_C \gg E_{e1}$$

$$N_{tr} = \rho_e^{2d} (F_C - E_{e1}) = \frac{m_e^*}{\pi \hbar^2 L_z} (F_C - E_{e1})$$

Large $N_{tr} \rightarrow$ High threshold current

(b) $m_h^* = m_e^*$ (Ideal semiconductor)

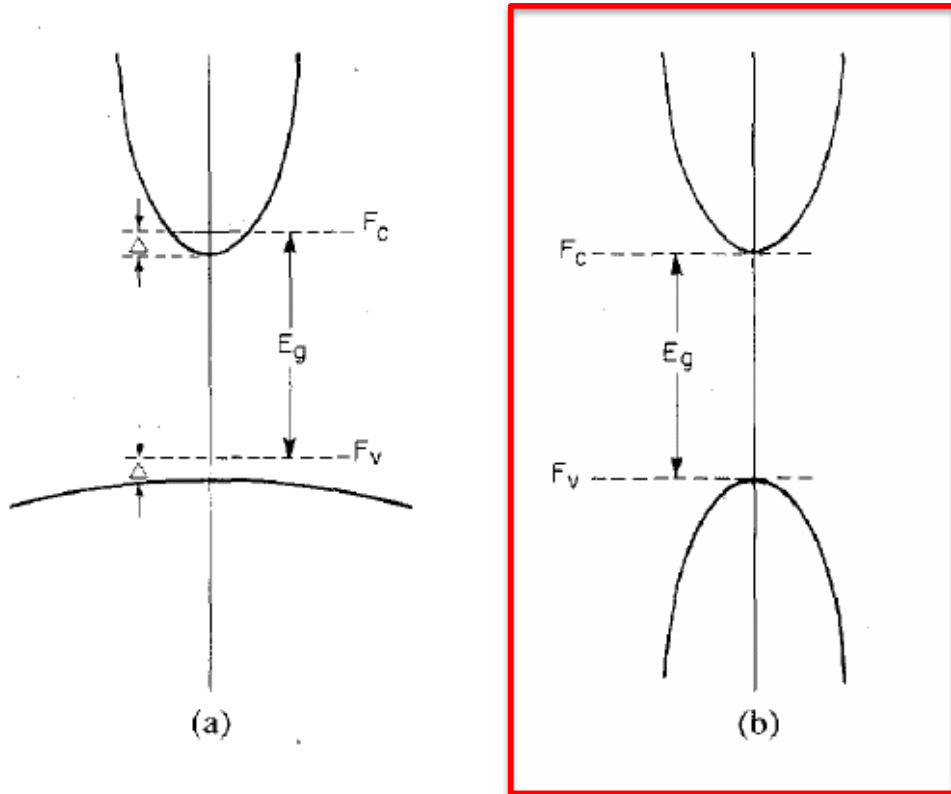
$$F_V = E_{h1}$$

$$F_C = E_{e1}$$

$$N_{tr} = \frac{m_e^*}{\pi \hbar^2 L_z} \int_{E_{e1}}^{\infty} f_C(E) dE \quad \text{is low}$$



Transparency Carrier Concentration for “Ideal” Quantum Well



Transparency Condition:

$$F_C - F_V = E_{e1} - E_{h1}$$

(b) Ideal Semiconductor

$$m_h^* = m_e^* \Rightarrow \boxed{F_V = E_{h1} \quad F_C = E_{e1}}$$

$$N_{tr} = \frac{m_e^*}{\pi^2 L_z} \int_{E_{e1}}^{\infty} \frac{1}{1 + e^{\frac{E - E_{e1}}{k_B T}}} dE$$

$$= \frac{k_B T m_e^*}{\pi^2 L_z} \int_0^{\infty} \frac{1}{1 + e^x} dx$$

$$= \frac{k_B T m_e^*}{\pi^2 L_z} \left(-\ln(1 + e^{-x}) \right) \Big|_0^{\infty}$$

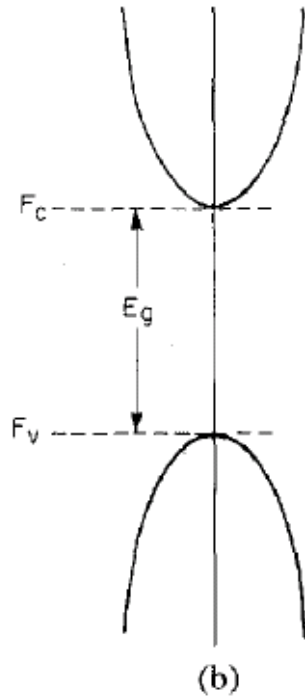
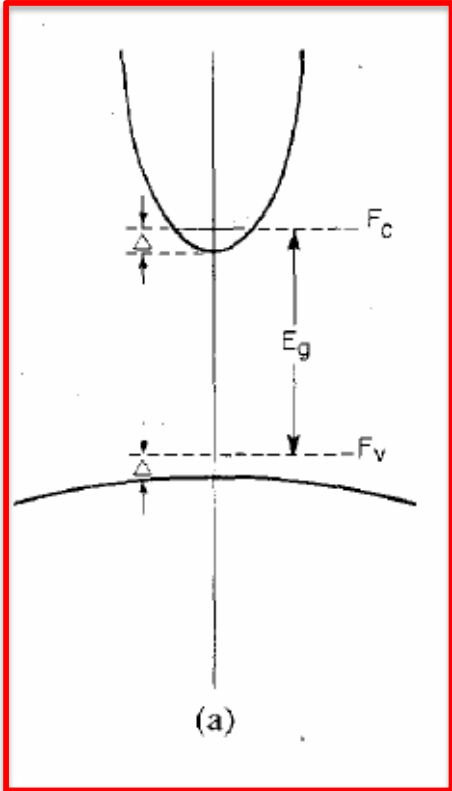
$$= \frac{k_B T m_e^*}{\pi^2 L_z} \ln 2$$

For $m_e^* = 0.067 m_0$

$$N_{tr} \approx 4.6 \times 10^{17} \text{ cm}^{-3}$$



Transparency Carrier Concentration for Normal Quantum Well



(a) Ordinary Semiconductor

$$N_{tr} = \rho_e^{2d} (F_C - E_{e1}) = \frac{m_e^*}{\pi^2 L_z} \Delta$$

To estimate Δ , note that $N = P$

$$P = N_V^{2d} e^{\frac{-\Delta}{k_B T}} = \frac{k_B T m_h^*}{\pi^2 L_z} e^{\frac{-\Delta}{k_B T}}$$

$$N = P \Rightarrow e^{\frac{-\Delta}{k_B T}} = \frac{\Delta}{k_B T} \frac{m_e^*}{m_h^*}$$

Transparency Condition:

$$F_C - F_V = E_{e1} - E_{h1}$$

For $m_h^* \approx 6m_e^*$ (in $1.55\mu\text{m}$ laser),

$$\Delta = 1.43 k_B T$$

$$N_{tr} = 1.43 \frac{k_B T m_e^*}{\pi^2 L_z}$$



Effective Mass Asymmetry Penalty

$$\frac{N_{tr}^{Ordinary}}{N_{tr}^{Ideal}} = \frac{1.43}{\ln 2} = 2$$

Threshold current density reduction is more than a factor of 2:

$$J_{th} = J_{nonrad} + J_{rad} + J_{Auger}$$

$$\frac{J_{th}}{qd} = AN + BN^2 + CN^3 = \frac{N}{\tau} + BN^2 + CN^3$$

τ : Shockley-Read-Hall nonradiative recombination lifetime

J_{Auger} is greatly reduced when N is lowered

(1) N^3 is reduced by 8x

(2) C is also reduced due to band structure change by strain



Bandgap-vs-Lattice Constant of Common III-V Semiconductors

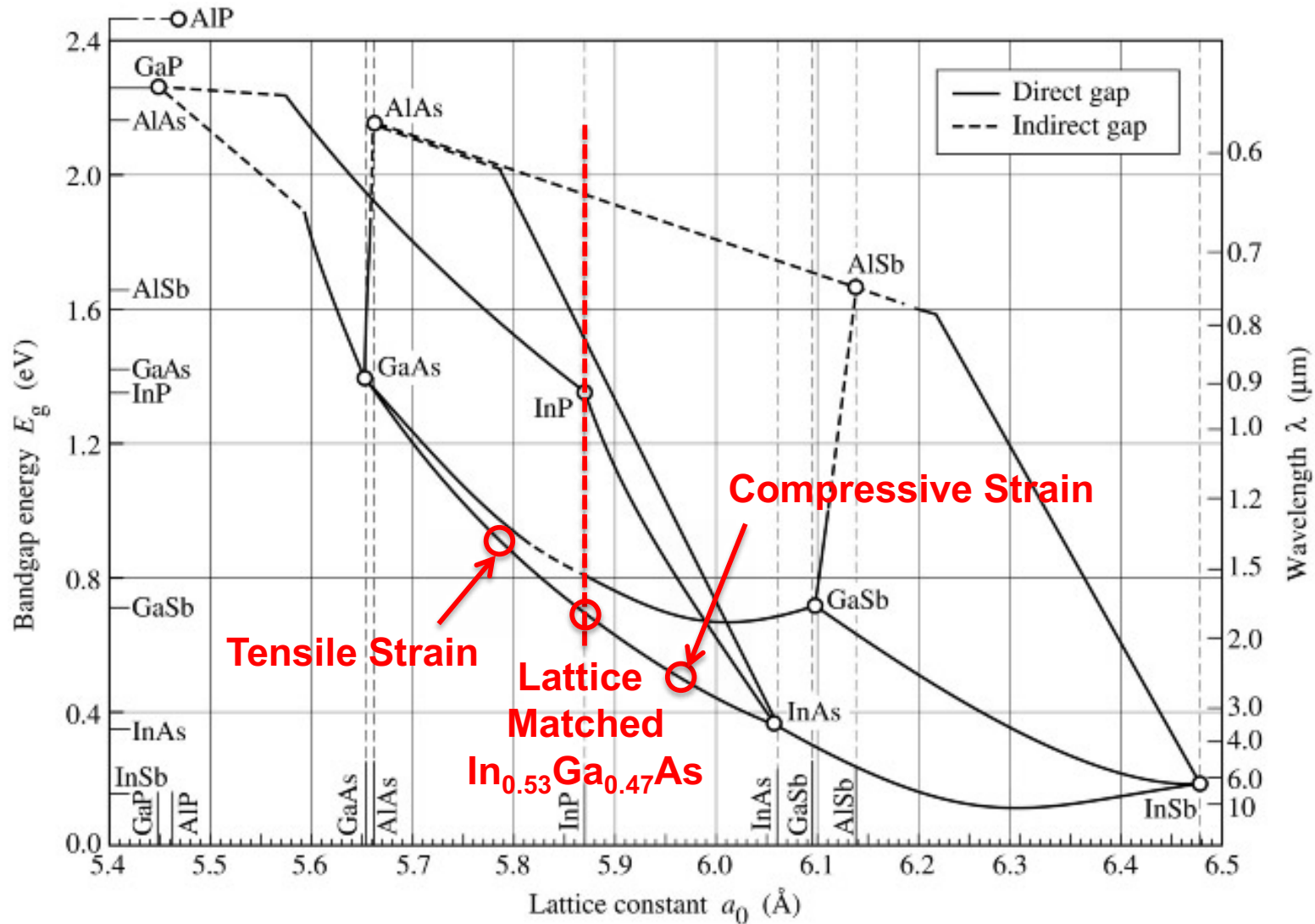
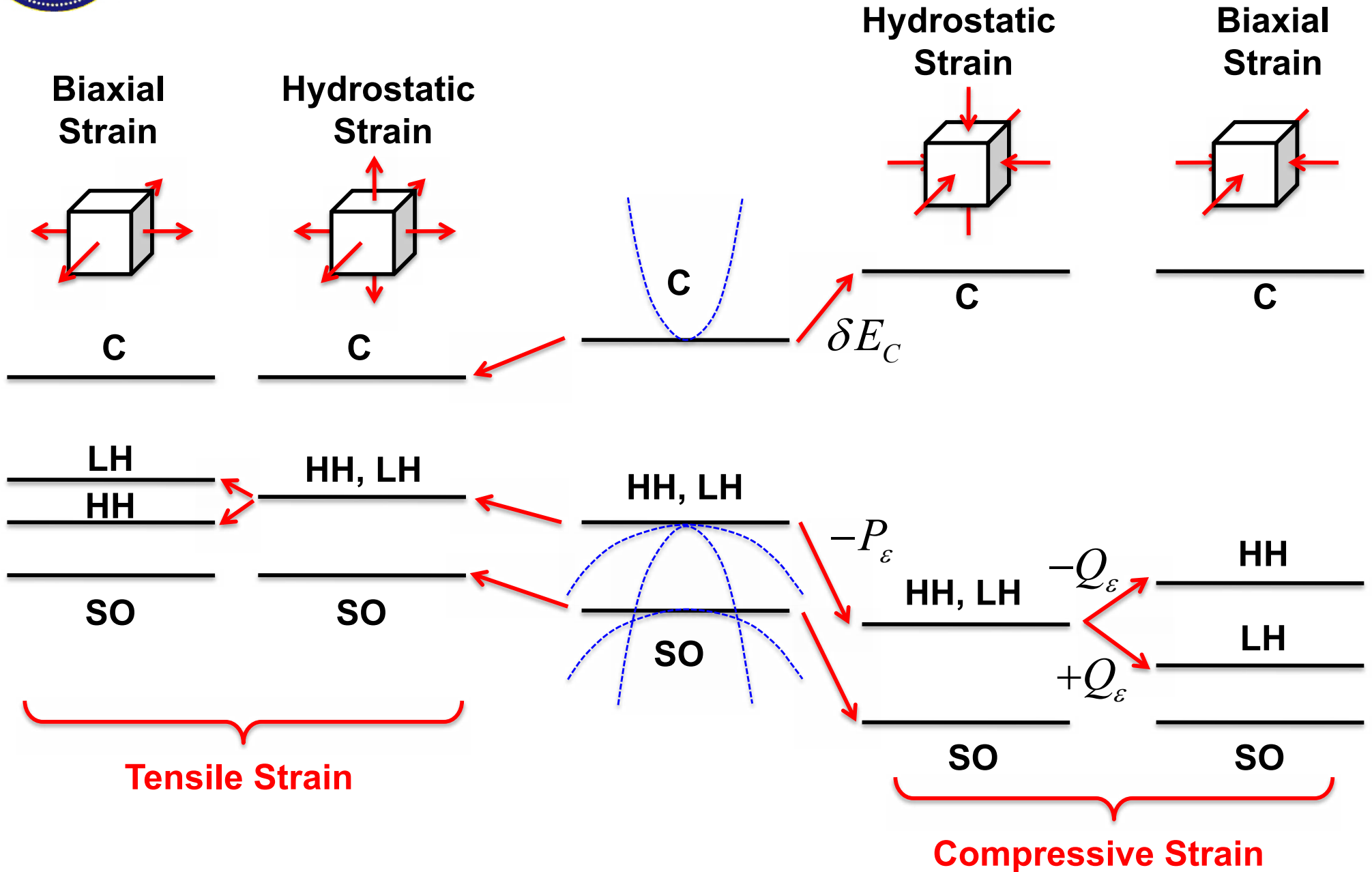


Fig. 7.6. Bandgap energy and lattice constant of various III-V semiconductors at room temperature (adopted from Tien, 1988).



Qualitative Band Energy Shifts Under Strain





Strain and Stress

$$\varepsilon = \varepsilon_{xx} = \varepsilon_{yy} = \frac{a_0 - a(x)}{a_0}$$

a_0 : lattice constant of InP

$\begin{cases} \varepsilon < 0 : \text{compressive strain} \\ \varepsilon > 0 : \text{tensile strain} \end{cases}$

$$\varepsilon_{\perp} = \varepsilon_{zz} = -2 \frac{C_{12}}{C_{11}} \varepsilon$$

C_{ij} : Compliance Tensor

$$C_{12} \approx 0.5C_{11}$$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} \\ C_{12} & C_{11} & C_{12} \\ C_{12} & C_{12} & C_{11} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{bmatrix}$$

Biaxial stress:

$$\sigma_{xx} = \sigma_{yy} = \sigma$$

$$\sigma_{zz} = 0$$

$$\Rightarrow C_{12}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{11}\varepsilon_{zz} = 0$$

$$\varepsilon_{zz} = -2 \frac{C_{12}}{C_{11}} \varepsilon$$



Band Edge Shift

$$E_C = E_g(x) + \delta E_C$$

$$E_{HH} = -P_\varepsilon - Q_\varepsilon$$

$$E_{LH} = -P_\varepsilon + Q_\varepsilon$$

$$\delta E_C = a_C (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = 2a_C \left(1 - \frac{C_{12}}{C_{11}} \right) \varepsilon$$

$$P_\varepsilon = -a_V (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = -2a_V \left(1 - \frac{C_{12}}{C_{11}} \right) \varepsilon$$

$$Q_\varepsilon = -b \left(\frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \varepsilon_{zz} \right) = -b \left(1 + 2 \frac{C_{12}}{C_{11}} \right) \varepsilon$$

$a = a_C - a_V$: hydrostatic potential

b : shear potential



Strain Parameters in III-V

(Coldren, p.535)

TABLE A11.1 Strain Parameters in III–V Semiconductors.

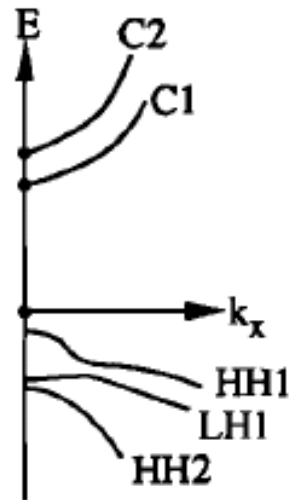
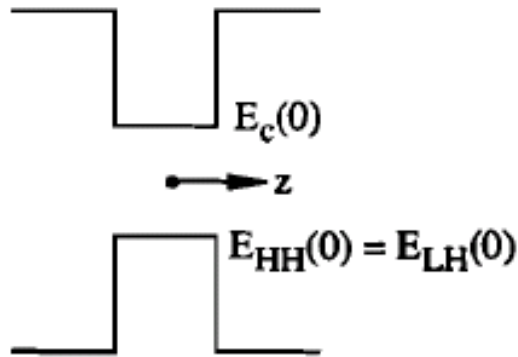
Material	Lattice Constant	Deformation Potentials (eV)			Elastic Moduli (10^{11} dyn/cm ²)			(10 ⁻⁶ eV/bar)	
	$a(\text{\AA})$	a	b	d	C_{11}	C_{12}	C_{44}	dE/dP	$\Delta(\text{eV})$
GaAs	5.6533	-8.68	-1.7	-4.55	11.88	5.38	5.94	11.5	0.34
InAs	6.0583	-5.79	-1.8	-3.6	8.329	4.526	3.959	10.0	0.371
AlAs*	5.6611	-7.96	-1.5	-3.4	12.02	5.70	5.89	10.2	0.30
GaP*	5.4512	-9.76	-1.5	-4.6	14.12	6.253	7.047	11.0	0.10
InP	5.8688	-6.16	-2.0	-5.0	10.22	5.76	4.60	8.5	0.10
AlP*	5.4635	-8.38	-1.75	-4.8	13.2	6.3	6.15	9.75	0.10
GaSb	6.0959	-8.28	-1.8	-4.6	8.842	4.026	4.322	14.7	0.8
InSb	6.4794	-7.57	-2.0	-4.8	6.47	3.65	3.02	16.5	0.98
AlSb*	6.1355	2.04	-1.35	-4.3	8.769	4.341	4.076	-3.5	0.75

* Indirect gap.

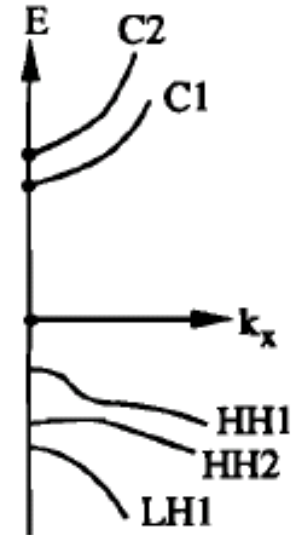
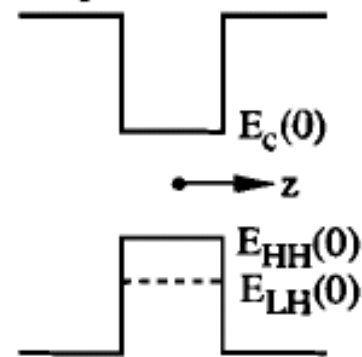


Band-Edge Profile and Subband Dispersion

(b) An unstrained quantum well



(a) A quantum well under a compressive strain



(c) A quantum well under a tensile strain

